A model for the multiwavelength radiation from tidal disruption event Swift J1644+57

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ABSTRACT

Gamma-ray observations of a stellar tidal disruption event (TDE) detected by the Swift satellite and follow up observations in radio, mm, infrared and x-ray bands have provided a rich data set to study accretion onto massive blackholes, production of relativistic jets and their interaction with the surrounding medium. The radio and x-ray data for TDE Swift J1644+57 provide a conflicting picture regarding the energy in relativistic jet produced in this event: x-ray data suggest jet energy declining with time as $t^{-5/3}$ whereas the nearly flat lightcurves in radio and mm bands lasting for about 100 days have been interpreted as evidence for the total energy output increasing with time. We show in this work that flat lightcurves don't require addition of energy to decelerating external shock (which produced radio and mm emission via synchrotron process), instead the flat behavior is due to inverse-Compton cooling of electrons by x-ray photons streaming through the external shock; the higher x-ray flux at earlier times cools electrons more rapidly thereby reducing the emergent synchrotron flux, and this effect weakens as the x-ray flux declines with time.

Key words: radiation mechanisms: non-thermal - methods: analytical - x-rays, radio: bursts, theory

1 INTRODUCTION

When a star wanders too close to a massive blackhole it is shredded by the strong tidal gravity of the BH. In this process a fraction of the star is captured by the BH and is eventually accreted, and roughly half of the stellar mass is flung out on hyperbolic orbits (Lacy et al. 1982; Rees 1988; Evans & Kochanek 1989; Goodman & Lee 1989; Ayal, Livio & Piran 2000). Such an encounter is expected to occur at a rate of 10^{-3}—10^{-5} yr^{-1} per L_* galaxy (Magorrian & Tremaine 1999; Wang & Merritt, 2004; Bower, 2011). The accretion disk is expected to produce blackbody radiation in UV and soft x-ray bands from the region close to the BH, and a bright optical flash is produced by a super-Eddington outflow and by the irradiation and photo-ionization of unbound stellar debris (Strubbe & Quataert 2009, 2010).

Generation of a relativistic jet is also expected to accompany a tidal disruption event (TDE) as the stellar material bound to the BH gravity is accreted over a period of time; we know empirically that accreting BHs in active galactic nuclei (AGNs) produce jets moving at speed close to that of light with Lorentz factor of order 10. The jet launched in a TDE interacts with the circum-nuclear

medium (CNM) and produces synchrotron radiation over a broad frequency band from radio to infrared which Giannios & Metzger (2011) predicted should be observable for several years (also Metzger et al. 2012).

The NASA Swift satellite discovered a transient event, Swift J1644+57, in γ & x-ray bands two years ago (Burrows et al. 2011, Bloom et al. 2011) which was subsequently observed in radio, mm, infrared and x-ray bands for a time period of more than year with good temporal coverage, e.g. Zauderer et al. (2011), Levan et al. (2011), Berger et al. (2012), Saxton et al. (2012), Zauderer et al. (2012), and the data is broadly in line with expectations of a TDE, e.g. Burrows et al. (2011).

Two other transient events observed in the last year have also been suggested to be TDE (Cenko et al. 2011, Gezari et al. 2012). However, the follow up observations for these events have been sparse, and so we consider in this paper only Swift 1644+57 and the puzzle posed by the rich data set for this event.

The mm & radio lightcurves for Sw J1644+57 were found to be flat/rising for $\sim 10^2$ days by Berger et al. (2012). These authors suggested that this implies continuous injection of energy to the decelerating blast wave, which produced this radiation, for $\sim 10^2$ d. However, that requires about 20 times more energy in the

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blast wave[1] than the initial jet energy that produced the strong X-ray signal which was of order 10^{52}erg. This energy requirement of more than a few times 10^{53}erg is very difficult to satisfy for a TDE model. We show in this paper that the multi-wavelength lightcurves of Sw J1644+57 can be understood without requiring energy in the blast wave to increase with time.

In the next section we describe the main features of the data for Swift 1644+57, and the main puzzles. We then describe in §3 the main idea of this paper that offers a simple solution to the puzzling data, and the application to Sw J1644+57 is provided in §4. Some concluding remarks about this event can be found in §5.

2   SWIFT J1644+57: SUMMARY OF OBSERVATIONS AND PUZZLES THEY HAVE POSED

Swift/XRT found that the x-ray flux from Sw J1644+57 varied rapidly on time scale $\delta t_{obs} \sim 10^2$s, indicating that the emission was probably produced at a distance of order $2c\delta t_{obs}\Gamma_j^2 \lesssim 10^{15}$cm; $\Gamma_j \lesssim 10$ is jet Lorentz factor. The average x-ray flux was roughly constant for the first 5 days, then dropped suddenly by a factor 50 during the next 3 days, and subsequently for $20d \lesssim t_{obs} \lesssim 500d$ it declined as $\sim t_{obs}^{-5/3}$ (Burrows et al. 2011; Berger et al. 2012; Zauderer et al. 2012). The x-ray flux then declined further by a factor ~ 170 between 500 days and 600 days (Zauderer et al. 2012). The spectrum in the 0.3–10 keV Swift/XRT band is reported to be a powerlaw function — $f_\nu \propto \nu^{-\beta}$ — with index $\beta \sim 0.7$ during the first ~ 50days and then decreasing to ~ 0.5 a few hundred days after the tidal disruption.

This transient event received extensive coverage at multiple mm and radio frequencies. Depending on the frequency band the lightcurve was found to rise, or remained nearly flat, during the first 100 days, and then declined as roughly t_{obs}^{-1}. The spectrum at low frequencies ($\lesssim 10^{10}$Hz) rose as $\sim \nu^{1.5}$ which is roughly consistent with that expected of a synchrotron spectrum below the self-absorption frequency (ν_a). And the spectrum above the peak of f_ν fell off as $\sim \nu^{-0.7}$. The synchrotron self-absorption frequency (ν_a) underwent a dramatic decline by a factor of 10 between 5 and 10 days, and subsequently ν_a displayed a very slow decline over the next several hundred days. The flux at the peak of the spectrum fell off during 5–10 days by a factor ~ 3, then increased by a factor ~ 4 during 10–10^2 days (Berger et al. 2012). It should be noted that these peculiar behavior of the radio data during the period 5–10 days coincided with a sharp decline of the x-ray flux by a factor $\sim 10^2$, e.g. Burrows et al. (2011), Berger et al. (2012).

The event was also observed in the infrared K-band ($\sim 1.4 \times 10^{14}$Hz) multiple times between 5 and 50 days, and the flux was found to decline during the first 5 days as $\sim t_{obs}^{-3}$ (Levan et al. 2011).

One of the main puzzles posed by the rich data for this event is that lightcurves in $\gtrsim 10$GHz frequency bands, which lie above the synchrotron absorption and other characteristic frequencies, show a flat or a rising behavior when in fact according to the external forward shock model these lightcurves should be declining as $\sim t_{obs}^{-1}$, e.g. Meszaros & Rees (1994), Chevalier and Li (2000), Panaitescu

& Kumar (2000), Granot and Sari (2002). Energy injection to the decelerating external blast wave between 10 and 10^2 days by a factor ~ 20 has been invoked, e.g. Berger et al. (2012), to prevent the decline of the radio lightcurve during this time interval. This huge amount of energy added to the blastwave, however, leaves no x-ray foot print, and that clearly is very puzzling[2]. The model we suggest — IC cooling of electrons by x-ray photons — avoids this problem because it requires no energy addition to the decelerating external shock to flatten radio/mm lightcurves.

Two other puzzles suggested by the data are the precipitous decrease of ν_a by a factor 10 between 5 and 10 days, and the rapid fall off of the infrared lightcurve during the same period.

3   INFLUENCE OF X-RAYS ON EXTERNAL SHOCK RADIATION

Basic idea: relativistic jet from the TDE interacts with the CNM (circum-nuclear medium) and drives a strong shock wave into it[3]. CNM electrons accelerated by the shock radiate synchrotron photons at radio and higher frequencies. These electrons are cooled by the IC scattering of x-ray photons streaming through the shocked plasma[4] (these x-ray photons are the same radiation that was observed by the Swift satellite); see Fig. 1 for a schematic sketch of this scenario. Since the x-ray flux declines rapidly with time ($f_x \propto t^{-5/3}$ for $t \gtrsim 10$ days), the IC cooling of electrons also diminishes with time and this is what is responsible for transforming an otherwise declining lightcurve ($f_\nu \gtrsim t_{obs}^{-1}$) to a flat or a rising lightcurve in radio/mm band; the IC cooling by x-rays turns out to be very effective in suppressing synchrotron radiation in mm/cm bands at early times, and this suppression gets weaker with time as the x-ray flux declines.

One other thing we should address here for those who might be thinking that the observed spectrum above the peak ($f_\nu \gtrsim \nu^{-0.8}$) is too shallow to be consistent with a fast electron cooling model. The observed parameters of Swift 1644+57 are such that IC scatterings for electrons radiating above the peak takes place in the Klein-Nishina regime, and hence the shallow observed spectrum; detailed calculations are presented in §3.1 & 3.2.

Since the main component of our proposed idea to explain radio/mm lightcurve behavior is IC cooling of electrons in the external shock by the observed x-ray radiation, we begin the technical part of this section with a description of this process. We show that it is unavoidable that IC cooling by x-ray photons significantly modifies electron distribution in the external shock for Sw J1644+57. We also discuss whether IC scatterings take place in the Thomson or Klein-Nishina (K-N for short, from now on) regime.

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[1] Using simple relativistic equipartition arguments Barniol Duran & Piran (2013) have shown that in the context of a standard synchrotron scenario, without IC cooling of electrons, the requirement that the blast wave energy should increase with time during the first ~ 200 days in order to explain the radio lightcurve behavior, does not depend on the choice of the geometry of the jet, nor does it depend on the precise spectral fitting of the radio data.
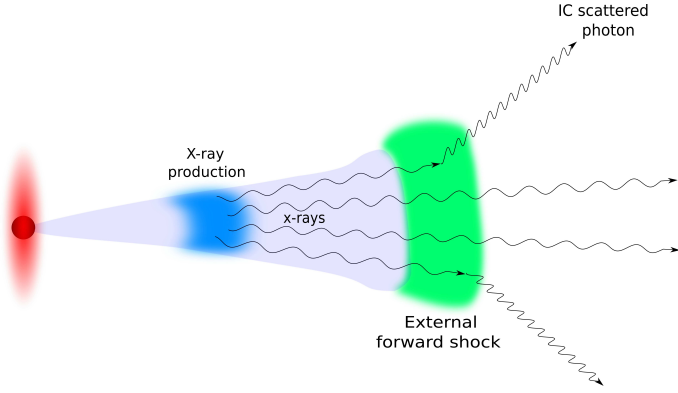
[2] The x-ray lightcurve fell off as $t_{obs}^{-5/3}$ for $t_{obs} \gtrsim 5$d, and therefore according to the x-ray data most of the energy release in relativistic jet took place during the first 5 days.

[3] A reverse shock propagating into the jet can also be produced in the CNM–jet interaction provided that the jet is not Poynting flux dominated, i.e. the ratio of energy flux carried by magnetic fields and the kinetic energy of matter (σ) is not much larger than 1. The mechanism described in this paper can also be applied to reverse shock emission. However, the external reserve-shock has difficulties explaining the observed flat radio/mm lightcurves.

[4] X-rays observed by the Swift satellite must have passed through the external shock region as long as they are produced at the same, or smaller, radius as the external shock. The very high variability of the x-ray data – on timescale of order minutes — suggests that the x-ray radiation is likely produced at a distance much smaller than the external shock radius.

Figure 1. Shown here is a schematic sketch of a jet produced in TDE. The energy is converted into x-rays at some distance from the black-hole by a as yet unknown mechanism. A part of this x-ray radiation is scattered by electrons in the forward-external shock which results in their efficient cooling at early times.

Finally, we calculate electron distribution function when IC cooling due to an external x-ray radiation field occurs in the K-N regime.

3.1 Importance of IC cooling and its effect on electron distribution

Let us consider a x-ray radiation source with luminosity $L_x(t_{obs})$ (isotropic equivalent) in frequency band ν_1–ν_2 which is produced by a source of size r_x. Its specific luminosity scales as $L_\nu \propto \nu^{-\beta}$; all unprimed variables are quantities measured in the rest frame of the host galaxy of the source. Now consider electrons in external forward shock at a radius $R(t_{obs})$, and the Lorentz factor of the shocked plasma (wrt CNM rest frame) to be $\Gamma(t_{obs})$. A photon of frequency ν moving at an angle θ wrt the radial direction is Doppler shifted to frequency ν' in shock comoving frame:

$$\nu' = \nu\Gamma(1 - \beta\cos\theta) \approx \frac{\nu}{2\Gamma}\left[1 + \theta^2\Gamma^2\right], \tag{1}$$

where

$$\beta = \left(1 - \Gamma^{-2}\right)^{1/2}, \tag{2}$$

and we used Taylor expansion for $\cos\theta$, and $\Gamma \gg 1$ to arrive at the approximate expression for ν'. From hereon all primed variables denote shock comoving frame.

The specific intensity in the host galaxy rest frame is

$$I_\nu(\theta) \approx \frac{L_\nu}{4\pi^2 r_x^2}, \tag{3}$$

for $\theta \lesssim r_x/R \equiv \theta_x$, and is zero otherwise. The specific intensity in the comoving frame of the shocked fluid is obtained by an appropriate Lorentz transformation, e.g. Rybicki & Lightman (1985), and is given by

$$I'_{\nu'} = I_\nu(\nu'/\nu)^3 \sim I_\nu/(8\Gamma^3), \tag{4}$$

when $\theta \ll \Gamma^{-1}$. Therefore, the specific flux in the comoving frame is

$$f'_{\nu'} \sim (\pi\theta'^2_x)I'_{\nu'} \sim \frac{L_\nu}{8\pi\Gamma R^2}, \tag{5}$$

where $\theta'_x \sim 2\Gamma\theta_x \ll 1$ is the angular size of the x-ray source as seen in the shock comoving frame. And the frequency integrated flux is

$$f'_x \sim \frac{L_x}{16\pi\Gamma^2 R^2}. \tag{6}$$

The factor Γ^2 in the denominator is easy to understand as it is due to Lorentz transformation of energy density.

The equation for IC cooling of an electron of Lorentz factor γ'_e is

$$\frac{d(m_e c^2 \gamma'_e)}{dt'_{ic}} = -\frac{4}{3}\sigma_T f'_x \gamma'^2_e, \tag{7}$$

where σ_T is Thomson scattering cross section. At the moment we are ignoring K-N effect which we will take up shortly. Thus, IC cooling time is given by

$$t'_{ic} \sim \frac{m_e c^2}{\sigma_T f'_x \gamma'_e} \sim (6 \times 10^6 s) R^2_{17} L^{-1}_{x,47}(\Gamma^2/\gamma'_e), \tag{8}$$

where we made equation of equation (6) for f'_x, and have adopted the widely used notation $X_n \equiv X/10^n$. The IC cooling calculation assumed that the external shock region is optically thin for x-ray photons. This condition is easily satisfied since the optical depth to Thomson scattering is $\sim \sigma_T n_e R \sim 10^{-7} n_e R_{17} \ll 1$ as long as n_e, the particle density in the unshocked CNM, is less than 10^7cm^{-3} at a distance of 10^{17}cm from the blackhole which is indeed the case as shown by radio observations. Also, absorption of x-ray photons by the inverse-Synchrotron process is negligibly small.

The dynamical time in the comoving frame is $t'_{dy} \sim R/(c\Gamma)$. Therefore,

$$\frac{t'_{ic}}{t'_{dy}} \sim \frac{2\Gamma^3 R_{17}}{\gamma'_e L_{x,47}}. \tag{9}$$

The Lorentz factor of the external shock a week after the tidal disruption is about 2 (see §3.2 for shock dynamics), $R_{17} \sim 1$, and $L_x \sim 5\text{x}10^{46}\text{erg/s}$. Thus we see from the above equation that the IC cooling time for electrons with $\gamma'_e \gtrsim 30$ is shorter than the dynamical time. Radio and mm photons observed from Sw J1644+57 are produced via synchrotron process by electrons with $\gamma'_e \gtrsim 50$, and so IC cooling cannot be ignored in the calculation of external shock radiation from this event. It should be pointed out that the synchrotron cooling is less important than the IC cooling at least for those electrons that are radiating well below the infrared band[5].

For electrons with $\gamma'_e \lesssim 10^2$ the IC scatterings take place in the Thomson regime for Sw J1644+57. The reason is that the energy of a 10 keV photon — Swift/XRT energy band is 0.3–10 keV — in the electron comoving frame is $(10\gamma'_e/\Gamma)$ keV which is less than $m_e c^2$ as long as $\gamma'_e \lesssim 10^2$. Higher energy electrons scatter 10 keV photons with a reduced cross-section[6], and the scattered photon energy is less than it is in the Thomson regime, i.e. $< \nu'\gamma'^2_e$. These two effects reduce the IC cooling of electrons with $\gamma'_e \gtrsim 10^2$. A simple, and fairly accurate, way to include these effects in IC cooling calculations is by only considering energy density in radiation field up to a photon energy of ϵ'_{kn}:

$$\epsilon'_{kn} \sim m_e c^2/\gamma'_e \quad \text{or} \quad \nu'_{kn} = \epsilon'_{kn}/h, \tag{10}$$

[5] The ratio of IC and synchrotron cooling times, in Thomson scattering regime, is equal to the ratio of energy density is magnetic field and radiation field. The x-ray luminosity from Sw J1644+57 was very high for the first several weeks, and thus it turns out that the energy density in radiation is much larger than magnetic energy density as long as ϵ_B for the shocked fluid is smaller than 1.

[6] Klein-Nishina cross-section is smaller than the Thomson cross section by a factor of photon energy in electron rest frame divided by $m_e c^2$.

where h is Planck's constant. The spectral index in the x-ray band for Swift J1644+57 is $\beta \approx 0.7$, and in that case the reduction to the IC cooling due to K-N effect scales as $\gamma_e'^{-0.3}$ for large γ_e' (see §3.1.1). This is a weak effect but has important implications for electron distribution function which is discussed below.

3.1.1 Electron distribution function in external shock

Electron distribution function, $dn_e/d\gamma_e$, in a shocked plasma is determined by the dual effect of acceleration at the shock front and IC and synchrotron coolings while electrons travel down-stream. The distribution function is obtained from the following equation:

$$\frac{\partial}{\partial t}\frac{dn_e}{d\gamma_e} + \frac{\partial}{\partial \gamma_e}\left[\dot{\gamma}_e\frac{dn_e}{d\gamma_e}\right] = S(\gamma_e), \qquad (11)$$

where

$$\dot{\gamma}_e = -\frac{4\sigma_T}{3m_ec^2}\left[f(<\nu_{kn}) + \frac{B^2c}{8\pi}\right]\gamma_e^2\beta_e^2, \qquad (12)$$

is the rate of change of γ_e due to IC and synchrotron losses, $f(<\nu_{kn})/c$ is energy density in radiation below the K-N frequency ν_{kn} (eq. 10), and

$$S(\gamma_e) \propto \gamma_e^{-p} \quad \text{for } \gamma_e \geqslant \gamma_i \qquad (13)$$

is the rate at which electrons with LF γ_e are injected into the system; γ_i is the minimum Lorentz factor for shock accelerated electrons. All the variables considered in this subsection are in shock comoving frame. However, we will NOT use prime (') on variables in this sub-section (and only in this sub-section) to denote shock comoving frame as this is the only frame being considered here. When results from this sub-section are used elsewhere in the paper we will revert back to the proper notation for prime and un-prime frames.

Let us define radiative cooling time

$$t_c(\gamma_e) = \frac{\gamma_e}{\dot{\gamma}_e}, \qquad (14)$$

and cooling Lorentz factor, γ_c, which is such that

$$t_c(\gamma_c) \equiv t_{dy}, \qquad (15)$$

where t_{dy} is the dynamical time.

We will calculate the distribution function in two cases. The slow cooling case where $\gamma_c > \gamma_i$, and the fast cooling regime when $\gamma_c < \gamma_i$. Moreover, we will consider here only IC losses in K-N regime as that is relevant for the "afterglow" radiation of the TDE Swift 1644+57. We note that the theoretical lightcurves and spectra shown for this event in the next section are obtained by numerical solutions of relevant equations that include synchrotron loss and no assumption regarding K-N regime.

Since $\nu_{kn} \propto \gamma_e^{-1}$ (eq. 10) and $f(<\nu_{kn}) \propto \gamma_e^{-(1-\beta)}$, therefore $\dot{\gamma}_e \propto \gamma_e^{1+\beta}$. We see from equation (11) that for $\gamma_e < \gamma_i$

$$\frac{dn_e}{d\gamma_e} \propto \frac{1}{\dot{\gamma}_e} \qquad (16)$$

whereas for $\gamma_e > \gamma_i$

$$\frac{dn_e}{d\gamma_e} \propto \frac{\int_{\gamma_e}^{\infty} d\gamma S(\gamma)}{\dot{\gamma}_e}. \qquad (17)$$

Therefore, the solution of equation (11) in the slow cooling case is

$$\frac{dn_e}{d\gamma_e} \propto \begin{cases} \gamma_e^{-p} & \gamma_i \leqslant \gamma_e \leqslant \gamma_c \\ \gamma_e^{-p-\beta} & \gamma_e > \gamma_c \end{cases} \qquad (18)$$

and in the fast cooling case

$$\frac{dn_e}{d\gamma_e} \propto \begin{cases} \gamma_e^{-1-\beta} & \gamma_c \leqslant \gamma_e \leqslant \gamma_i \\ \gamma_e^{-p-\beta} & \gamma_e > \gamma_i \end{cases} \qquad (19)$$

The synchrotron spectrum for particle distribution $dn_e/d\gamma_e \propto \gamma_e^{-q}$ is $f_\nu \propto \nu^{-(q-1)/2}$ for $\nu > \min(\nu_c, \nu_i)$, and it is $\nu^{1/3}$ for $\nu < \min(\nu_c, \nu_i)$; these spectra are modified when synchrotron-absorption becomes important (the criterion for this is discussed in the next sub-section)[7].

It should be clear from these equations that the spectral index above the peak depends on p, and the spectral index β of the radiation field that is responsible for IC cooling of electrons; the spectral index for $\nu > \max(\nu_i, \nu_c)$ is $(p - 1 + \beta)/2$ which is harder than $\nu^{-p/2}$. As an application to Swift J1644+57 we note that the spectral index in the radio band above the peak of the spectrum is reported to be ~ 0.75 for $t_{obs} \gtrsim 10^2$ d (Berger et al. 2012). This does not mean that $p = 2.5$ as suggested by eg. Berger et al. If IC cooling in K-N regime is important for determining particle distribution, which equation (9) for t_{ic}'/t_{dy}' makes clear is the case, then $p = 2.3$, since according to the late time X-ray data $\beta = 0.4$ (Saxton et al. 2012).

3.2 Shock dynamics and synchrotron lightcurves

The dynamics of external forward shock can be determined, approximately, from the following energy conservation equation

$$\frac{4\pi}{(3-s)}R^3 R_{17}^{-s} n_f m_p c^2 \Gamma(\Gamma - 1) = E, \qquad (20)$$

where E is the energy (isotropic equivalent) in the blast wave, the density of the CNM is taken to be

$$n(R) = n_f R_{17}^{-s}, \qquad (21)$$

and $R_{17} = R/10^{17}$ cm. For a relativistic shock, i.e. $\Gamma \gg 1$, the dynamics is described by

$$\Gamma = \left[\frac{(3-s)E}{4\pi m_p c^2 n_f R^3 R_{17}^{-s}}\right]^{1/2}, \qquad (22)$$

and the deceleration radius – where the relativistic ejecta has imparted roughly half its energy to the surrounding medium – is

$$R_{d,17} = \frac{(3-s)E_{51}}{4\pi m_p c^2 n_f \Gamma_0^2}, \qquad (23)$$

where Γ_0 is the initial Lorentz factor of the relativistic outflow. For a non-relativistic shock the solution of equation (20) is

$$v = \left[\frac{(3-s)E_{51}}{2\pi m_p n_f R_{17}^{3-s}}\right]^{1/2}. \qquad (24)$$

The Lorentz factor for the external shock of TDE Swift 1644+57 was of order a few at early times (less than a week), and the shock speed decreased to Newtonian regime at late times. Hence the analytical calculations we present in this section considers both of these regimes. The numerical results described in the next section are more accurate and do not rely on making any assumption regarding Γ.

A photon released at radius R arrives at the observer at time

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[7] A very detailed description of synchrotron spectrum when IC cooling in K-N regime is important can be found in Nakar et al. (2009)

$$t_{obs} = \frac{(1+z)}{c} \int dR \left[\frac{c}{v} - 1 \right]. \tag{25}$$

For relativistic blast waves $t_{obs} \propto R^{4-s}$, and for non-relativistic case $t_{obs} \propto R^{(5-s)/2}$. Moreover, the shock speed as viewed by an observer is

$$\Gamma \propto t_{obs}^{-\frac{3-s}{8-2s}} \quad \text{and} \quad R \propto t_{obs}^{1/(4-s)}, \tag{26}$$

for the relativistic case and

$$v \propto t_{obs}^{-\frac{3-s}{5-s}} \quad \text{and} \quad R \propto t_{obs}^{2/(5-s)}, \tag{27}$$

for Newtonian dynamics.

The deceleration time in observer frame, for relativistic outflow, is given by

$$t_d \approx (3 \times 10^4 \text{s}) \left[\frac{(3-s)E_{53}}{1.8\, n_{f,2}} \right]^{\frac{1}{3-s}} (1+z)\Gamma_{0,1}^{-\frac{8-2s}{3-s}}. \tag{28}$$

Thus, for $E = 10^{53}$erg, $n_f = 10^2$cm^{-3}, and $\Gamma_0 = 10$, the deceleration time is of order 5 hours, and the Lorentz factor of the shocked CNM 10 days after the start of TDE is ~ 2.

3.2.1 External shock radiation and lightcurves

We provide analytical calculation for synchrotron radiation and lightcurves from external forward shock in this sub-section; more accurate numerical results are presented in the next section.

The magnetic field strength and the electron minimum Lorentz factor in a shock heated plasma are estimated by assuming that a fraction of the energy of shocked plasma (ϵ_B) goes into the generation of magnetic fields, and a fraction (ϵ_e) is taken up by electrons[8]. Under this assumption we find that

$$B' = \left[32\pi\epsilon_B m_p c^2 n(R)\Gamma(\Gamma-1) \right]^{1/2}, \tag{29}$$

and the minimum Lorentz factor of shock accelerated electrons is

$$\gamma_i' = \frac{(p-2)m_p}{(p-1)m_e}\epsilon_e(\Gamma-1) \tag{30}$$

Using results for shock dynamics from last sub-section we find

$$B' \propto \begin{cases} t_{obs}^{-\frac{3}{2(4-s)}} & \Gamma\beta \gg 1 \\ t_{obs}^{-\frac{3}{5-s}} & \Gamma\beta \ll 1 \end{cases} \tag{31}$$

and the synchrotron frequency corresponding to electron Lorentz factor γ_i' is

$$\nu_i = \frac{qB'\gamma_i'^2\Gamma}{2\pi m_e c} \propto \begin{cases} t_{obs}^{-3/2} & \Gamma\beta \gg 1 \\ t_{obs}^{-\frac{15-4s}{5-s}} & \Gamma\beta \ll 1 \end{cases} \tag{32}$$

We next calculate synchrotron frequency for those electrons that cool on a dynamical timescale (ν_c) due to IC scattering of x-ray photons. Let us rewrite equation (7) to explicitly consider x-ray flux below the K-N frequency

$$\frac{dm_e c^2 \gamma_e'}{dt_{ic}'} = -\frac{4}{3}\sigma_T f_x'(< \nu_{kn}')\gamma_e'^2, \tag{33}$$

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where ν_{kn}' is given by equation (10). The LF of electrons that cool on a dynamical time is obtained from the above equation

$$\gamma_c' \sim \frac{3m_e c^2}{4\sigma_T f_x'(< \nu_{kn}')t_{dy}'}. \tag{34}$$

For a powerlaw spectrum for x-ray radiation

$$f_x'(< \nu_{kn}') = \frac{L(< \nu_{kn})}{16\pi\Gamma^2 R^2} \propto \frac{L_x(t_{obs})}{R^2\Gamma^{1+\beta}\gamma_c'^{1-\beta}}. \tag{35}$$

Therefore, the electron "cooling" Lorentz factor is given by

$$\gamma_c' \propto \left[\frac{R^2\Gamma^{1+\beta}}{t_{dy}'L_x(t_{obs})} \right]^{1/\beta}, \tag{36}$$

Using equations for shock dynamics from last sub-section we find

$$\gamma_c' \propto \begin{cases} \left[\dfrac{t_{obs}^{[2-(2+\beta)(3-s)]/[2(4-s)]}}{L_x(t_{obs})} \right]^{1/\beta} & \text{for } \Gamma\beta \gg 1 \\[4mm] \left[\dfrac{t_{obs}^{(s-1)/(5-s)}}{L_x(t_{obs})} \right]^{1/\beta} & \text{for } \Gamma\beta \ll 1 \end{cases} \tag{37}$$

The synchrotron frequency in observer frame corresponding to γ_c' is

$$\nu_c = \frac{qB'\gamma_c'^2\Gamma}{2\pi(1+z)m_e c^2}. \tag{38}$$

Or

$$\nu_c \propto \begin{cases} t_{obs}^{-3/2} \left[\dfrac{t_{obs}^{(s-2)/(4-s)}}{L_x(t_{obs})} \right]^{2/\beta} & \text{for } \Gamma\beta \gg 1 \\[4mm] t_{obs}^{-3/(5-s)} \left[\dfrac{t_{obs}^{(s-1)/(5-s)}}{L_x(t_{obs})} \right]^{2/\beta} & \text{for } \Gamma\beta \ll 1 \end{cases} \tag{39}$$

For $s = 1.5$, $\beta = 0.7$ and $L_x \propto t_{obs}^{-5/3}$ we find $\nu_c \propto t_{obs}^{2.7}$ for relativistic case, and $\nu_c \propto t_{obs}^{4.3}$ for Newtonian dynamics. For these parameters for Sw J1644+57, ν_c is about 1 GHz at 10 days, and it rises to 100 GHz at ~ 70 days.

The observed flux at a frequency $\nu > \max(\nu_a, \nu_i, \nu_c)$ is given by

$$f_\nu(t_{obs}) = f_p \nu_c^{\beta/2} \nu_i^{(p-1)/2} \nu^{-(p+\beta-1)/2}, \tag{40}$$

where ν_a is synchrotron absorption frequency,

$$f_p \approx \frac{q^3 B'\Gamma N_e}{m_e c^2} \frac{1+z}{4\pi d_L^2}, \tag{41}$$

$N_e \propto R^{3-s}$ is the total number of electrons with Lorentz factor $> \min(\gamma_i', \gamma_c')$, and d_L is the luminosity distance to the source; we made use of equations (18) and (19) in deriving equation (40).

Making use of equations (26), (27), (31), (32) and (39) in the above expression for f_ν we find the observed lightcurve behavior:

$$f_\nu(t_{obs}) \propto \frac{\nu^{-(p+\beta-1)/2}}{L_x(t_{obs})} \times$$
$$\begin{cases} t_{obs}^{-[2+3\beta+3(p-1)]/4} & \Gamma\beta \gg 1 \\[3mm] t_{obs}^{\frac{19-6s-p(15-4s)-3\beta}{[2(5-s)]}} & \Gamma\beta \ll 1 \end{cases} \tag{42}$$

For $p = 2.0$, $\beta = 0.7$, $s = 1.5$ and $L_x \propto t_{obs}^{-5/3}$, $f_\nu \propto \nu^{-0.85}t_{obs}^{-0.1}$ for relativistic dynamics, and $f_\nu \propto \nu^{-0.85}t_{obs}^{0.2}$ for Newtonian shock. The dynamics of Swift J1644+57 for $10d \lesssim t_{obs} \lesssim 10^2$d lies

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[8] This crude approximations is made due to our inability to calculate magnetic fields and electron acceleration in collisionless shocks ab initio. And the dimensionless parameters ϵ_B and ϵ_e hide our ignorance regarding these processes.

between these two extremes, and the lightcurve is flat for as long a duration as the observing stays above ν_c. Of course, at sufficiently high frequencies — which of order 10^{13} Hz for this TDE at ~ 10d — electron cooling by IC scattering becomes unimportant due to K-N suppression, and synchrotron cooling time becomes smaller than the dynamical time. In this case the lightcurve decline has the well known form of $f_\nu \propto \nu^{-p/2} t_{obs}^{-(3p-2)/4}$ for relativistic shocks, e.g. Kumar (2000).

The rapid increase of ν_c with time is responsible for the nearly flat lightcurve observed for Swift J1644+57 in frequency bands $\nu > \max(\nu_a, \nu_i, \nu_c)$. When $\nu_c(t_{obs})$ moves above the observing band then the lightcurve resumes its usual, $\sim t_{obs}^{-1}$, decline which occurs at $t_{obs} \gtrsim 10^2$ days for $\nu \gtrsim 10^2$ GHz as shown in numerically calculated lightcurves presented in the next section.

Another interesting case to consider is when $\nu_c < \nu < \nu_i$ and the observing band is above the synchrotron absorption frequency. The lightcurve in this case is determined from

$$f_\nu(t_{obs}) = f_p \nu^{-\beta/2} \nu_c^{\beta/2}. \tag{43}$$

Making use of equations (41) and (39) we find

$$f_\nu(t_{obs}) \propto \frac{\nu^{-\beta/2}}{L_x(t_{obs})} \begin{cases} t_{obs}^{-(2+3\beta)/4} & \Gamma\beta \gg 1 \\ t_{obs}^{[4-2s-3\beta]/[2(5-s)]} & \Gamma\beta \ll 1 \end{cases} \tag{44}$$

For $\beta = 0.7$, $s = 1.5$ and $L_x \propto t_{obs}^{-5/3}$ the lightcurve rises as $\nu^{0.35} t^{0.64}$ ($t_{obs}^{1.5}$) for relativistic (Newtonian) blast waves.

Next, we calculate the synchrotron self-absorption frequency (ν_a) when IC cooling is important. The easiest way to determine ν_a is by using Kirchhoff's law or the fact that the emergent flux at frequency ν_a is approximately equal to the black-body flux corresponding to electron temperature given by

$$kT_e' = \max[\gamma_a', \min(\gamma_i', \gamma_c')]m_e c^2, \tag{45}$$

where γ_a' is electron Lorentz factor corresponding to synchrotron frequency ν_a', i.e. $\gamma_a' \propto (\nu_a'/B')^{1/2}$. The equation for ν_a', using Kirchhoff's law, is

$$\frac{2kT_e'\nu_a'^2}{c^2} \approx \frac{\sigma_T B' m_e c^2 N_e(> kT_e/m_e c^2)}{4\pi q}, \tag{46}$$

where the left side of this equation is the Planck function in the Rayleigh-Jeans limit, and $N_e(> kT_e/m_e c^2)$ is the column density of electrons with LF larger than $\max[\gamma_a', \min(\gamma_i', \gamma_c')]$.

One case of particular interest for application to Sw J1644+57 is where ν_a is larger than $\max(\nu_i, \nu_c)$. We can rewrite equation (46) for this case as

$$2m_e \nu_a'^{5/2} \left(\frac{2\pi m_e c}{qB'}\right)^{1/2} = f'(\nu_a') = \frac{f_{\nu_a} d_L^2}{\Gamma R^2}. \tag{47}$$

Making use of equations (26), (27), (31) and (42) we find

$$\nu_a \propto \begin{cases} t_{obs}^{-\frac{8+(4-s)(3p+3\beta+2)}{2(4-s)(p+\beta+4)}} \Big/ L_x^{\frac{2}{p+\beta+4}} & \Gamma\beta \gg 1 \\ t_{obs}^{-\frac{30-6s-p(15-4s)-3\beta}{(5-s)(p+\beta+4)}} \Big/ L_x^{\frac{2}{p+\beta+4}} & \Gamma\beta \ll 1 \end{cases} \tag{48}$$

For $s = 1.5$, $p = 2$, $\beta = 0.7$, and $L_x \propto t_{obs}^{-5/3}$, $\nu_a \propto t_{obs}^{-0.49}$ ($t_{obs}^{0.45}$) for relativistic (Newtonian) shock dynamics. We note that due to the rapid rise of ν_c (see eq. 39) the above equation for ν_a ceases to be applicable after a few 10s of days for Sw J1644+57

when the ordering of synchrotron characteristic frequency changes to $\nu_i < \nu_a < \nu_c$. The synchrotron absorption frequency for this case can be shown to be

$$\nu_a \propto \begin{cases} t_{obs}^{-\frac{1}{p+4}\left[\frac{10-s}{2(4-s)} + \frac{3(p-1)}{2}\right]} & \Gamma\beta \gg 1 \\ t_{obs}^{-\frac{[(5+4s)+(15-4s)(p-1)]}{(5-s)(p+4)}} & \Gamma\beta \ll 1 \end{cases} \tag{49}$$

For $s = 1.5$ and $p = 2$, $\nu_a \propto t_{obs}^{-0.53}$ ($t_{obs}^{-0.95}$) for relativistic (Newtonian) shock dynamics.

The rise of lightcurve for about 10^2 days for $\nu \lesssim 10$GHz is due to the fact that these observing bands lie below the synchrotron absorption frequency during this period. The observed flux below ν_a decreases rapidly with decreasing ν, and hence even a slow decrease of ν_a with time leads to a rising lightcurve.

We note that the luminosity in high energy photons produced by the IC scattering of x-rays by electrons in the external shock is of order $L_x Y$, where $Y \sim \sigma_T n(R)R\gamma_c'\gamma_i' \sim 10^{-3} R_{17} n_{f,2}$ is the Compton parameter, and the energy of these scattered photons is ~ 10MeV.

4   NUMERICAL RESULTS AND APPLICATION TO SWIFT J1644+57

Equations (20) & (25) are solved numerically to determine shock dynamics as a function of observer time, and equations (11), (12), (29), (30) & (46) are used for calculating spectra and lightcurves that take into account electron cooling due to IC scattering of an "external" radiation field with flux and spectrum taken from x-ray observations, and also cooling due to synchrotron radiation. No assumption regarding whether IC scatterings are in K-N or Thomson regime are made for calculations presented in this section.

The energy in the blast wave is injected only at the beginning of numerical calculation when the blast wave is at a radius $R = 10^{13}$cm; results are insensitive to the precise value of the initial radius as long as it is taken to be much smaller than the deceleration radius. In Fig. 2 we show R, Γ, B', γ_i & γ_c, for parameters relevant for Sw J1644+57.

Theoretically computed lightcurves for Sw J1644+57 in frequency bands 230 GHz & 87 GHz, together with observed data from Berger et al. (2012), are shown in Fig. 3. The theoretical lightcurves were calculated from a decelerating blastwave which had a fixed amount of total energy (i.e. no late time injection of energy) and yet the observed flux in these frequency bands is nearly flat for $\sim 10^2$ days (consistent with the observed data) in spite of the fact that these bands clearly lie above the synchrotron injection (ν_i) and self-absorption (ν_a) frequencies (shown in fig. 4). There is a noticeable dip in lightcurves between ~ 10 & 20 days. The reason for this dip is that the observed x-ray lightcurve during this period is nearly flat (except, of course, for the ever present fluctuations). A flat x-ray lightcurve means that ν_c does not increase during this time interval (see fig. 4). However, ν_i and the peak flux (which are independent of L_x) continue their monotonic decline, and that is the reason for the flux decrease during this period. A more realistic treatment of the x-ray flux in our calculation, that includes the observed fluctuations, should provide a better fit to the observed data.

The nearly flat lightcurves for $\sim 10^2$ days for 87 and 230 GHz is due to the fact that these bands lie above the synchrotron cooling frequency which rises very rapidly with time ($\nu_c \propto t_{obs}^3$) due to the rapidly declining x-ray flux; we obtained this behavior analytically
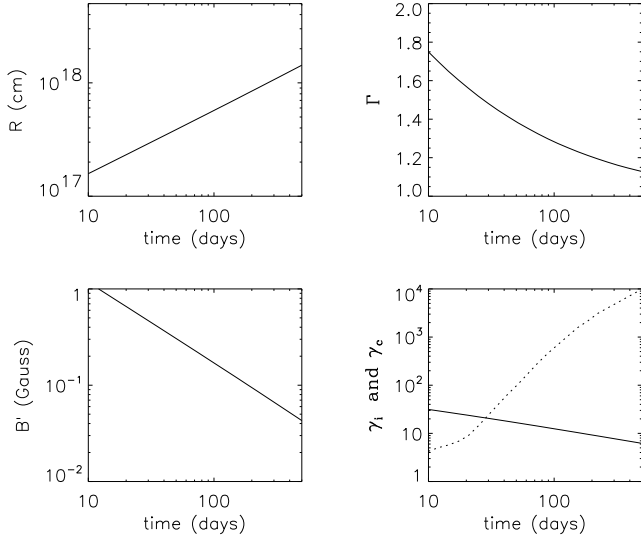
Figure 2. A few of the parameters of the external forward show are shown in this figure as a function of observer time. Shock front radius (top left), Lorentz factor of the shocked fluid (top right), magnetic field in the comoving frame of shocked fluid (bottom left), and bottom right panel shows γ_i (solid line) & γ_c (dotted line). The energy in the blast-wave was taken to be $E = 1.5 \times 10^{53}$ erg (isotropic), the CNM density $n(R) = (400\text{cm}^{-3} \times R_{17}^{-2}$, $\epsilon_B = 4.5 \times 10^{-3}$, $\epsilon_e = 0.1$, and $p = 2.3$ for these calculations. The x-ray luminosity was taken to be $L_x = 4 \times 10^{46}$ erg/s$(t_{obs}/20\text{days})^{-5/3}$ for $t_{obs} > 20$d and constant between 10 and 20 days; x-ray photons passing through the external shock cooled electrons due to IC scatterings and that was the dominant cooling mechanism for electrons radiating at radio/mm frequencies for $\sim 10^2$ days. We note that the uncertainty in γ_c is small as it depends primarily on L_x which is well determined from observations. However, γ_i might be much larger than shown in this figure, particularly for $p \approx 2$ favored by our modeling of the observed data, if the minimum Lorentz factor of electrons accelerated in shocks is larger than the value we have adopted in this paper, viz. $\gamma_i = \epsilon_e(p-2)(\Gamma - 1)m_p/[m_e(p-1)]$.

in the last section, and the numerical result is presented in fig. 4. The lightcurve remains nearly flat as long as the observing band is above ν_c, and then it starts to decline as $\sim t_{obs}^{-0.8}$. The duration of the flat phase is dictated by the temporal behavior of x-ray flux which we took for these calculations to decline as $t_{obs}^{-5/3}$, and the spectrum was taken to be a powerlaw function; $f_\nu \propto \nu^{-0.7}$ for $0.1\text{keV} \leqslant \nu \leqslant 10\text{keV}$ as per Swift/XRT observations[9] for $t_{obs} < 40$ days, eg. Saxton et al. (2012). The observed x-ray lightcurve in reality is much more complicated, and shows rapid fluctuations. However, we don't expect ν_c, and radio lightcurves, to follow these wild fluctuations because the signal we receive from the external shock at any given fixed observer time is in fact an integral over equal arrival time 3-D hypersurface that has a large radial width so that the effect of x-ray radiation on electron distribution function is

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[9] Swift/XRT observes photons with energy between 0.3 and 10 keV. However, it would be a remarkable coincidence if the spectrum for Sw 71644+57 starts deviating from the observed powerlaw behavior exactly at the low or the high energy threshold for XRT. The infrared k-band flux at 10 days was 0.03 mJy (Levan et al. 2011) and at the same time the mean 1 keV flux was 0.02 mJy (Zauderer et al. 2012), and that suggests that the XRT spectrum became much harder, perhaps $\nu^{1/3}$, at ~ 0.1 keV. The flux above 10 keV, however, is rather poorly constrained by observations.
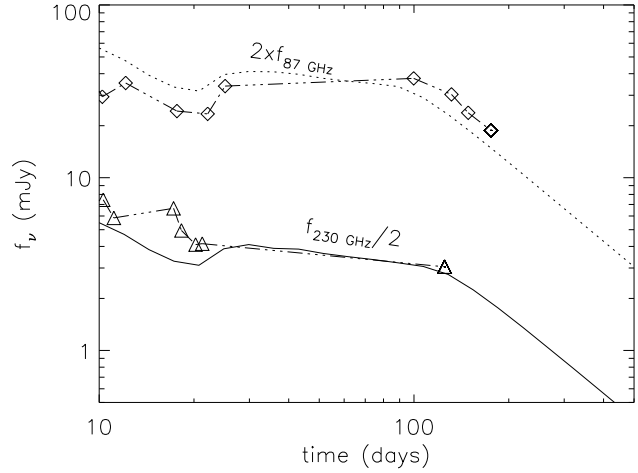
Figure 3. Lightcurves in frequency bands 230 GHz (solid line) & 87 GHz (dotted curve) calculated for the model described in §3, together with observed data for Sw J1644+57 from Berger et al. (2012) are shown in this figure. The parameters used for these calculations are same as that in Fig. 2 (see fig. 2 caption for details). Lightcurve before 10 days is not shown here because the wild fluctuation in x-ray flux at earlier times (by more than a factor 10) introduce huge error in our theoretical calculations which is based on a powerlaw decline model for x-ray luminosity for $t_{obs} > 20$days, and a constant luminosity for 10d $\leqslant t_{obs} \leqslant 20$d.

averaged over a time duration δt_{obs} of order t_{obs}. The x-ray flux averaged over this time scale is not a smooth powerlaw function, but consists of periods of faster and slower declines. The duration of time when lightcurve in an observed frequency band is roughly flat depends on these fluctuations, and for that reason the duration of the flat phase shown in figure 3 should be considered to have an uncertainty of order 20%.

The lightcurve decline for frequency ν which lies below the synchrotron cooling frequency (ν_c) is easy to obtain using the scalings provided in the §3.2.1. For sub-relativistic shocks, $B' \propto t_{obs}^{-3/(5-s)}$ (eq. 31), and $\gamma_i \propto t_{obs}^{-(15-4s)/(5-s)}$ (eq. 32), and so the observed flux for $\nu_c > \nu > \max(\nu_a, \nu_i)$ decreases with time as $t_{obs}^{-[(15-4s)(p-1)-(6-4s)]/2(5-s)}$; for J1644+57, $\nu_c \sim 10$ GHz at $t_{obs} \sim 10^2$d (Fig. 4). For $s = 2$ and $p = 2.3$, the lightcurve decline is expected to be $t_{obs}^{-1.85}$ which is consistent with the observed decline (see. the 87 GHz lightcurve in Fig. 3 on this page, and also 8.4 — 43 GHz lightcurves in Fig. 3 of Zauderer et al. 2012).

The reason that we don't show theoretical lightcurves in fig. 3 for $t_{obs} < 10$ days is that the x-ray lightcurve before 10 days is dominated by very large flares — the flux decreased by a factor $\sim 10^2$ between 4 and 6 days — which cannot be captured by a simple powerlaw model used in this paper. A calculation of IC cooling of electrons, and the emergent flux, before 10 days must include these wild fluctuations for the result to be meaningful.

The rise in lightcurves at lower frequencies (10 GHz or less) and for 10d $< t_{obs} \lesssim 10^2$d is due to ν_a lying above the band (see Fig. 4). This behavior can be understood using analytical scalings presented in the last section. The above remark regarding fluctuations in $L_x(t_{obs})$ applies to ν_a as well, although self-absorption frequency is much less sensitive to L_x than ν_c is.

The solutions we are finding has $\min(\nu_i, \nu_c) < \nu_a$, and that might naively suggest that the spectrum below ν_a should be $\nu^{2.5}$ in conflict with the observed spectrum of $\sim \nu^{1.5}$ for $t_{obs} \lesssim 10^2$d. The fact that the observed spectrum is shallower than ν^2 is already
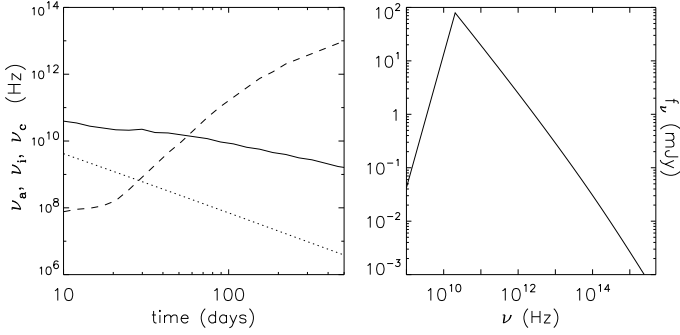
Figure 4. The left panel shows the three synchrotron characteristic frequencies, obtained by numerical calculations, for the same set of parameters as fig. 2; see fig. 2 caption for details; solid line is ν_a, dotted line is ν_i and the dashed curve is ν_c. Note the sharp rise in ν_c, as $\sim t_{obs}^3$, between 20 and 100 days, which is consistent with our analytical estimates (see §3). We note that the uncertainty in the calculation of ν_c is small. However, the actual value of ν_i might be larger than in this figure by 1–2 orders of magnitude for $p \approx 2$ favored by our modeling of the observed data (see caption for Fig. 2 for explanation); good fits to observed lightcurves (like those shown in fig. 3) can be obtained even when ν_i is much larger than shown in this figure by appropriately rescaling microphysics parameters and changing E and n by a factor ~ 2. The right side panel shows the spectrum calculated at 20 days. The spectrum above ν_i and ν_c is $\nu^{-0.8}$ even though we took $p = 2.3$. This is due to the fact that when cooling is dominated by IC scatterings of x-ray photons in the K-N regime, the spectrum is $\nu^{-(p-1+\beta)/2}$ instead of $\nu^{-p/2}$; the spectral index for x-ray radiation was $\beta = 0.4$ for $t_{obs} \gtrsim 50$d (Saxton et al. 2012). The spectrum steepens to $\sim \nu^{-1.15}$ for $\nu \gtrsim 10^{13}$Hz when the cooling is dominated by synchrotron loss.

telling us that something is missing. And most likely what that is, is that the jet has some angular structure, and when integration over equal arrival time hyper-surface is carried out to obtain the observed spectrum, it turns out to be softer by $\nu^{0.5}$—ν^1 than the comoving, instantaneous, spectrum[10].

The infrared flux at 10 days (0.03 mJy, Levan et al. 2011) is consistent with the theoretically calculated value for parameters used for fitting 230 GHz lightcurve shown in Fig. 3. The infrared producing electrons have $\gamma_e' \sim 10^4$ and their IC cooling by scattering x-rays photons is highly suppressed due to K-N effect. However, ν_c due to synchrotron cooling is $\sim 10^{13}$Hz, so the spectrum in this band should be $\sim \nu^{-1}$. Levan et al. (2011) report that the infrared lightcurve declined with time as $\sim t_{obs}^{-3}$ between 5 and 10 days. This is much too steep for external shock and we suspect that this might have something to do with the sharp drop in the x-ray flux between 4 and 7 days by a factor ~ 50.

A theoretically calculated spectrum for external shock synchrotron radiation at 52 days is shown in Fig. 4. The spectrum above the peak is quite hard, $\sim \nu^{-0.7}$, even though ν_i and ν_c lie

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[10] The equal arrival time hyper-surface is a 3-D section of the 4-D space–time with the property that radiation from every point on this hyper-surface arrives at the observer at the same time. The radius (R), γ_i, B' etc. for different points on the hypersurface are in general different for a non-uniform jet, i.e. a jet where energy density, Lorentz factor etc. vary with angle wrt jet axis and distance from the jet surface. It can be shown that an integration over equal arrival time hyper-surface invariably softens the observed spectrum.

below the peak. The reason is that when electrons are cooled by IC scatterings in K-N regime the resulting spectrum is $\nu^{-(p-1+\beta)/2}$ (see §3.1) and not $\nu^{-p/2}$. X-ray data show that β varies with time (Burrows et al. 2011), and that β was about 0.7 for $t_{obs} \lesssim 50$d and ~ 0.4 for $t \gtrsim 50$d (Saxton et al. 2012). Therefore, we expect the radio spectrum above the peak to be $\sim \nu^{-0.85}$ at early times, and $\nu^{-0.7}$ at late times, for $p = 2$. The late time radio data are consistent with this expectation, and for $t_{obs} \lesssim 50$d spectra above the peak are poorly sampled.

4.1 High energy radiation

IC scattering of an x-ray photon of frequency ν_x by an electron of Lorentz factor γ_i' in the external shock produces a high energy photon of energy $\sim 2\gamma_i'^2\nu_x$. Considering that $\gamma_i' \sim 30$ (Fig. 2), the IC photon energy is ~ 10 MeV, and the expected IC luminosity is $\sim \tau_T L_x \gamma_i' \min(\gamma_c', \gamma_i')$; where $\tau_T \sim \sigma_T n(R) R \sim 10^{-5} R_{17} n_{f,2}$ is Thomson optical depth. At $t_{obs} = 10$d, $L_x \sim 3 \times 10^{46}$erg s^{-1}, $\gamma_c' \sim 4R_{17}$ & $\gamma_i' \sim 30$ (fig. 2), and therefore we expect IC luminosity to be $\sim 5 \times 10^{43} R_{17}^2 n_{f,2}$ erg s^{-1}. Another way to estimate the luminosity of high energy IC photons follows directly from the fact that IC scatterings of x-ray photons is efficient at cooling electrons at early times, i.e. IC cooling time is less than or of order the dynamical time for $t_{obs} < 20$d. Hence, the IC luminosity should be roughly equal to the energy luminosity carried by electrons accelerated by the external shock, i.e. $L_{ic} \lesssim 16\pi\epsilon_e R^2 m_p c^3 n(R) \Gamma^2 \sim 2 \times 10^{44} \epsilon_{e,-1} n_{f,2} R_{17}^2$erg s^{-1}. There is no data between 1 & 100 MeV for Sw J1644+57 to our knowledge, however, Fermi/LAT provided an upper limit of about 10^{46}erg/s above 100 MeV at 10d (Burrows et al. 2011). This suggests that n_f, the CNM density at $R = 10^{17}$cm, cannot be larger than about 10^4cm^{-3}.

4.2 Parameters of the TDE and the external shock

An accurate determination of jet energy and other parameters for Sw J1644+57 should take into consideration the effect of fluctuating L_x on electron distribution, a more detailed treatment of the dynamics than described by equation (20), and the 3D structure of shocked plasma and magnetic fields. But that is beyond the scope of this work[11]. We show in Fig. 5 an approximate determination of E (isotropic energy in the external shock), the CNM density, and ϵ_B (energy fraction in magnetic fields) allowed by the multiwavelength data for Sw J 1644+57; see the caption for Fig. 5 for a description of the various observational constraints that were used for the determination of these parameters.

Fermi/LAT upper limit restricts the CNM density at 10^{17}cm to be smaller than $\sim 5 \times 10^3$cm^{-3} (see §4.1). And according to the radio/mm/infrared data the isotropic energy in the blastwave is $\sim 2 \times 10^{53}$erg which is consistent with estimates obtained using equipartition argument, e.g. Barniol Duran et al. (2013). We note that the uncertainty in these parameters and ϵ_B is a factor of a few. It should be pointed out that the afterglow data we have used for determining energy etc. consists of observations carried out 10 or more days after the TDE. The blastwave Lorentz factor at this time had dropped to $\lesssim 2$, and the angular size of the shocked CNM is

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[11] The goal of this work is to show that cooling of electrons by x-ray photons passing through the external shock can explain the puzzling, flat, lightcurves observed in mm/cm bands for $\sim 10^2$days.
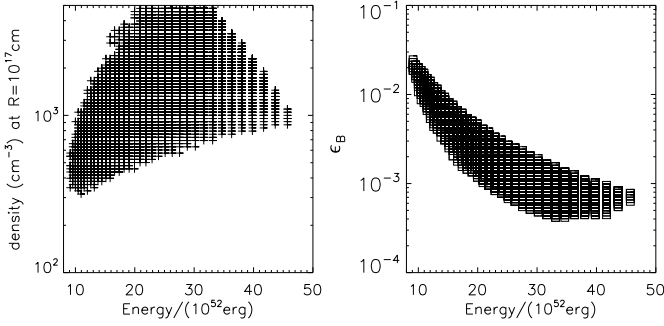
Figure 5. Isotropic blast wave energy (E), CNM density at $R = 10^{17}$ cm, and ϵ_B allowed by the data for Sw J1644+57. The parameters shown in this figure should be considered a rough estimate due to the approximate way we have handled the blastwave dynamics, and used a smooth decline for the x-ray flux passing through the external shock and cooling electrons instead of the highly fluctuating flux Swift/XRT observed. The observational constrains that were used in the determination of parameters are: (1) flux at 230 GHz at 10 days after the tidal disruption was 10 mJy (Berger et al. 2012), (2) $\nu_a = 15$ GHz at 10 days, (3) the flux at the peak of the spectrum at 10^2 days is 50 mJy, (4) an upper limit of 10^{46}erg s^{-1} for $>10^2$MeV at 10 days which suggests that $n_f \lesssim 10^4$cm^{-3} (n_f is density at $R = 10^{17}$cm); a tolerance of $\pm 30\%$ was used for all these constraints which reflects the degree of uncertainty of our calculations. We took $L_x = 4 \times 10^{46}$erg/s$(t_{obs}/20{\rm days})^{-5/3}$ for $t_{obs} > 20$d and constant between 10 and 20 days, and $\epsilon_e = 0.1$ for these calculations; for $\epsilon_e = 0.5$ these plots look very similar, except that they are shifted to the left by a factor ~ 10.

perhaps larger by a factor ~ 2 than the jet angle at the smaller radius where x-rays were produced. Therefore, the beam corrected energy in the blastwave is perhaps a factor ~ 2–4 larger than the energy in x-rays which suggests a $\gtrsim 20\%$ efficiency for converting jet luminosity to x-rays.

We have also looked for the density stratification (s) and find that $s \gtrsim 1.5$ provides a reasonable fit to the available data; $s = 1.5$ density stratification is expected for an advection dominated accretion flow in the neighborhood of a blackhole, and $s = 2$ is when outflow determines the CNM density structure.

Although we took $\epsilon_e = 0.1$ for results shown in all of the figures, other values of ϵ_e are allowed by the data as long as we take $E \propto \epsilon_e^{-1}$.

We find that flux in the infrared K-band at 10 days for the solution space shown in Fig. 5 is between 0.02mJy and 0.04 mJy; the observed K-band flux at this time was 0.03 mJy (the error in flux calculations, due to various simplifications we have made, is a factor $\lesssim 2$). The observed K-band lightcurve for $t_{obs} \lesssim 10$d decayed as $\sim t_{obs}^{-3}$ (Levan et al. 2011) which is too steep for external forward shock emission and we suspect that this might have something to do with large x-ray flares that occurred between 5 and 10 days (see §4.3 for further discussion).

The 1 keV x-ray flux at 610 days after Swift/BAT trigger — for the entire solution space shown in Fig. 5 — is within a factor ~ 2 of the value observed by the Chandra satellite which was 0.7nJy (Levan & Tanvir, 2012).

4.3 A puzzling drop in ν_a between 5 & 10 days

According to Berger et al. (2012) the synchrotron-absorption frequency (ν_a) dropped by a factor 10 between 5 and 10 days after the tidal disruption event Sw J1655+57. If these observations are correct[12] then this result essentially rules out any shock model, with or without continuous energy injection, for radio emission unless some mechanism external to the shock led to a drastic change to the electron energy and magnetic fields in shocked plasma between 5 and 10 days. To understand this result we make use of equation (46) for ν_a':

$$\nu_a'^2 \gamma_e'(\nu_a') \propto B' N_e(\geqslant \gamma_e') \text{ or } \nu_a \propto \Gamma \left[B'^{3/2} N_e(\geqslant \gamma_e') \right]^{0.4} \quad (50)$$

where $\gamma_e'(\nu_a')$ is the Lorentz factor of electrons that have characteristic synchrotron frequency ν_a', and $N_e(\geqslant \gamma_e')$ is the column density of electrons that radiate above ν_a'. The right hand side of the first equation is proportional to the synchrotron flux at ν_a' in the comoving frame of the source when synchrotron absorption is ignored, and in the second equation we have considered the case where $\nu_a' > \min(\nu_i', \nu_c')$ (other cases are straightforward to consider). For any external shock model, with or without energy injection and arbitrary CNM density stratification, Γ, B' and N_e are smooth functions of time (see §3). Since $\Gamma \propto t_{obs}^{-(3-s)/(8-2s)}$ (eq. 26), the most change we can expect for Γ between 5 and 10 days is a factor $2^{3/8} \sim 1.3$ for a $s = 0$ medium; similarly, $B' \propto \Gamma R^{-s/2}$ can't change much during this time in the absence of an external agency at work. And N_e, which depends on the number of electrons in the CNM swept up by the shock, is typically an increasing function of time and hence its effect on ν_a' is to increase it [13]. We know empirically that $N_e(\geqslant \gamma_e')B' \propto f(\nu_a)/\Gamma$ decreased by a factor ~ 3 between 5 and 10 days for Sw J1644+57 (see fig. 2 of Berger et al. 2012), and that by itself can only account for a decrease of ν_a by a factor $\lesssim 2$. We see, therefore, that unless there is a dramatic change in B' and/or $\gamma_e'(\nu_a')$ on a short time scale of $\delta t_{obs}/t_{obs} = 1$ — which is not possible in external shocks without a powerful radiation front passing though it or some such mechanism — ν_a cannot drop by a factor ~ 10 between 5 and 10 days that is observed for Sw 1644+57; any addition of energy to a decelerating shock only makes the problem worse.

It is probably no coincidence that the x-ray luminosity of Sw 1644+57 fell by a factor ~ 50 during the same time period that ν_a dropped by a factor 10. When a highly luminous x-ray front was passing through the external shock ($t_{obs} \lesssim 6$d), electrons in the region were cooled by IC scatterings of x-rays to a Lorentz factor of order unity in less than one dynamical time (see eq. 9). The sharp drop in x-ray luminosity during 6 to 8 days by a factor 50 reduced the IC cooling of electrons and raised γ_c' by the same factor. As we can see from equation (50) a rising electron temperature helps to lower ν_a, however, the effect on $\gamma'(\nu_a')$ is much smaller. Ultimately, a sharp drop in B' is needed to account for the observed decrease of ν_a as can be seen from the second part of equation (50). The rapid

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[12] We would not be surprised if it were to turn out that the drop in ν_a has been overestimated by observers by a factor 2–3. The spectrum reported by Berger et al. (2012) at $t_{obs} = 10$d has no frequency coverage between 15 and 80 GHz. It is possible that the actual peak was at ~ 50GHz, in which case ν_a at 10d has been underestimated by a factor ~ 3.

[13] If a shock were to run into a dense cloud, that would cause ν_a to increase. And when a shock suddenly encounters a zero density medium, the shocked plasma undergoes adiabatic expansion and then γ_e', B', and $N_e'(\geqslant \nu_a')$ decrease with time. It is easy to show that the net effect of adiabatic expansion is a slight decrease of ν_a in time interval t_{obs}—$2t_{obs}$.

fall off of the infrared lightcurve mentioned in §2 might also be due to this decline of magnetic field. We suspect that a sharp drop in L_x leads to a smaller B'. One possible way this might occur is that while L_x is large, or jet power is high, the interface separating the shocked jet and the shocked CNM is Rayleigh-Taylor unstable and the turbulence associated with this instability produces strong magnetic fields. Currents driven in the shocked CNM by high L_x might be another way to generate close to equipartition magnetic field in the region where radio/mm emission is produced. However, a connection between L_x and magnetic field generation is just a speculation at this point[14].

5   CONCLUDING REMARKS

An interesting puzzle is posed by the radio/mm data for the tidal disruption event Swift J1644+57: lightcurves in frequency bands that lie above the synchrotron characteristic peak frequencies, i.e. $\nu > 10$ GHz, are roughly flat for 10^2 days rather than decline with time as $\sim t_{obs}^{-1}$ as one would expect for radiation produced in the external shock. Berger et al. (2012) suggested that the flat lightcurves are due to continued addition of energy to the external shock for 10^2 days. This requires about 10 times more energy to be added at late times than is seen in x-rays, and that is worrisome. We have described in this paper a simple model for flat lightcurves that does not require adding any extra energy to the decelerating shock wave. The flat lightcurves, instead, are a result of efficient cooling of electrons in the external shock by IC scatterings of x-ray photons that pass through the external shock region on their way to us (Fig. 1). The IC cooling becomes weaker with time as the x-ray flux declines and that is the origin of radio/mm lightcurve flatness. This model can account for the radio, milli-meter, infrared and $>10^2$MeV data (temporal decay as well as spectra).

The angular size of the x-ray beam should not be much smaller than the size of the external shock region during the initial $\sim 10^2$ days for the IC cooling model to work; otherwise only a fraction of shock accelerated electrons will be exposed to IC cooling, and the effect described here is weakened. Considering the *naturalness* of this model — it is unavoidable that x-ray photons pass through the external shock and cool electrons — and its clear cut, robust, implications for external shock lightcurves, one could perhaps turn this restriction around and suggest (1) that the angular size of the x-ray producing part of the jet increases with time so that it is never much smaller than the angular size of the shocked CNM (2) **or** that the external shock does not undergo much sideways spreading for at least $\sim 10^2$ days even as its Lorentz factor falls with time to order unity[15]. The late time radio lightcurves ($t_{obs} \gtrsim 200$days) declined as $\sim t_{obs}^{-1.8}$ (Zauderer et al. 2012; Fig. 3) which is consistent with synchrotron radiation from a sub-relativistic blast-wave, and the jet spreading is poorly constrained in this case.

The model predicts a connection between the observed x-ray

lightcurve and the temporal and spectral properties of radio and mm radiation from Sw J1644+57. It requires more extensive numerical modeling than presented in this paper to explore this connection in detail, which can then be confronted with the extensive data for this remarkable event.

Barniol Duran & Piran (2013) have suggested two broad class of models for Sw J1644+57 using equipartition arguments within the context of synchrotron emission, but without considering the IC cooling of electrons. The first scenario consists of one jet that is responsible for both the X-ray and the radio emission. The second scenario invokes two sources: a narrow relativistic jet to produce X-rays, and a quasi-spherical, mildly relativistic, source to produce the radio emission. Both models require the energy available to electrons for synchrotron radiation to increase with time, but Barniol Duran & Piran (2013) argue that the total energy in the radio source could be constant. The model suggested in the present work roughly corresponds to the first scenario, with a constant total energy, where the energy fraction available to electrons for synchrotron radiation increases with time as the x-ray luminosity decreases and the IC losses weaken.

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[14] We don't expect the magnetic field to continue to decline with L_x indefinitely. The reason for this is that below a certain threshold x-ray luminosity magnetic field generation in external forward shock proceeds by Weibel mechanism, or some other instability, that is not affected by L_x.

[15] Kumar & Granot (2003) using 1-D hydro simulations found little evidence for sideways spreading for a beamed jet interacting with ISM as long as it was relativistic. More sophisticated 2-D simulations, e.g. van Eerten & MacFadyen (2012), find a similar result. However, jets appear to undergo sideways expansion, according to theoretical calculations, when their Lorentz factor drops to order unity.